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Numerical Modeling of Field Patterns in a Photonic Crystal Waveguide with Metamaterial Cylindrical Inclusions

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Abstract. The study of photonic crystal waveguides (PCWs) is of current interest from the theoretical and applied point of view of optics, telecommunications and even in the area of medicine in the monitoring of biomolecules. In this work we consider an electromagnetic system composed of two flat conductive plates and a periodic array of circular cylindrical metamaterial (LHM) inclusions forming a PCW. A numerical integral method was applied to determine the dispersion relation that allows us to characterize eigenmodes of the system and the intensity of field of its electromagnetic modes. The results show ordered and disordered patterns of field intensities in a dispersive LHM medium. Although the presence of a disordered pattern is not enough to affirm the presence of electromagnetic chaos phenomenon, this behavior can be evidenced through an analysis of the field in terms of spatial statistics and correlations, which allows us to show numerically that the correlation length of the field autocorrelation function approaches zero. These wave chaos properties in a PCW involving LHM could have various applications in modern communication technology.

Keywords: Photonic crystal waveguide, metamaterial, electromagnetic chaos, correlation length.

1 Introduction

The study of photonic crystal waveguides (PCWs) is of current interest from the theoretical and applied point of view of optics, telecommunications and even in the area of medicine in the monitoring of biomolecules [1]. A particularly important issue in this field is the attempt to identify evidence of chaos phenomenon in the transport properties of ballistic systems. As matter of fact, the magnetoresistance has been measured in chaotic and regular cavities showing clear distinctions in quantum transport [2].

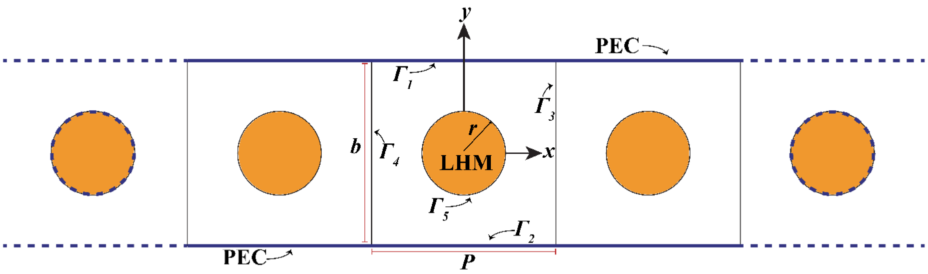

Fig. 1. Diagram of the PCW formed by two PEC flat surfaces and a periodic array of circular cylindrical inclusions of LHM in the x-direction.

The signature of chaos in classical transport through waveguides has also been investigated, and shows a completely different behavior on the resistivity when the system is regular or chaotic [4]. In this paper, the Integral Equation Method (IEM) [5] was applied to calculate and examine the electromagnetic field intensities of a PCW, formed by two perfect electric conductor (PEC) flat surfaces and a periodic array of circular cylindrical metamaterial (LHM) inclusions, which presents disordered patterns.

Despite the fact that the presence of a disordered pattern is not enough to confirm the presence of the electromagnetic chaos phenomenon, the corresponding classical channel, such as the Sinai billiards obviously exhibit chaotic behavior [7]. Thus, this behavior can be evidenced through an analysis of the field in terms of spatial statistics and correlations, which allows us to show numerically that the correlation length of the field autocorrelation function approaches zero.

A validation of this assumption has been provided by the investigations on the spatial autocorrelation functions that are much more sensitive to the nature of eigenmodes [3]. This paper is organized as follows. In Sec. 2 we introduce an integral method to calculate the field intensity in the unit cell of our system that includes LHM dispersive media associated with electromagnetic modes, based on the ideas outlined elsewhere [5, 6]. Section 3 shows numerical results for a wide range of frequencies, which present disordered patterns. Finally, Sec. 4 presents our conclusions.

2 Theoretical Approach

We consider an infinite two-dimensional system composed of two PEC flat surfaces with circular cylindrical LHM inclusions as shown in Fig. 1. Assuming a harmonic time dependence for TM polarized electromagnetic field, $\mathbf{H} = H(\mathbf{r},t)\exp(-i\,\omega\,t)\hat{\mathbf{k}}$, the wave equation for medium j becomes the Helmholtz equation:

$$\left[\nabla^2 + k_j^2\right] H_j(\mathbf{r}) = 0, \tag{1}$$

where $\mathbf{r}$ is the position vector in the xy-plane, and we define $k_j = n_j(\omega)(\omega/c)$ with $n_j(\omega) = \pm\sqrt{\varepsilon_j(\omega)\mu_j(\omega)}$ the refractive index of the j-th medium involving the optical properties of the material. The sign of the refractive index will be positive for a dielectric or real conductive medium and negative for LHM inclusions, which is given in terms of magnetic permeability an electric permittivity given by [5] $\varepsilon(\omega) = 1 - \omega_p^2/\omega^2$ and

$\mu(\omega) = 1 - f\omega_p^2/\left(\omega^2 - \omega_0^2\right)$, where ω_0 is the resonance frequency, ω_p is the plasma frequency and f is the filling fraction of the unit cell. The region where this LHM has a negative refractive index value is within the frequency range $\omega_0 << \omega << \omega_{LH}$ with $\omega_{LH} = \omega_0/\sqrt{1-f}$. Equation (1) can be represented in integral form for the medium j, considering as a solution the two-dimensional Green's function $G\left(\mathbf{r}, \mathbf{r}'\right)$ of the equation:

$$\left[\nabla^2 + k_j^2\right] G_j\left(\mathbf{r}, \mathbf{r}'\right) = -\delta\left(\mathbf{r}, \mathbf{r}'\right). \quad (2)$$

And the Green's integral theorem [5, 6]. Thus, the general form of the Helmholtz integral equation is obtained for the j-th medium:

$$\frac{1}{4\pi} \oint_{\Gamma_j} \left[G_j\left(\mathbf{r}, \mathbf{r}'\right) \frac{\partial H_j(\mathbf{r})}{\partial n'} - H_j\left(\mathbf{r}'\right) \frac{\partial G(\mathbf{r}, \mathbf{r}')}{\partial n'}\right] ds' = H_j(\mathbf{r}) \Theta(\mathbf{r}), \quad (3)$$

Being $G(R) = (i/4) H_0^1(kR)$, where $H_0^1(\zeta)$ is the Hankel function of the first kind and zero order, $R = |\mathbf{r} - \mathbf{r}'|$ and $\Theta(\mathbf{r}) = 1$ if $\mathbf{r}$ is inside the region j and $\Theta(\mathbf{r}) = 0$ otherwise. Due to the geometry of the study problem (see Fig. 1), we have the boundary conditions for the TM polarization on the contour Γ_j, $H_j = H_{j+1}$ and $1/\varepsilon_j \partial H_j/\partial n = 1/\varepsilon_{j+1} \partial H_{j+1}/\partial n$.

And by translational symmetry through a PCW, based on Bloch's theorem, we can state that $H(x - P, y) = H(x, y) \exp(-iKP)$ where K is the one-dimensional Bloch vector. In order to solve Eq. (3) numerically, it is necessary to make a discretization by dividing the curve Γ_j that defines the j-th region into curve segments of arc length Δs small enough so that the field and its normal derivative are constant. Therefore, the integrals of Eq. (3) for the j-th region can be approximated and converted into a system of linear equations [5, 6] for the linearly polarized magnetic field.

Then, the linear system can be represented by $F(K, \omega)\ M(K, \omega) = 0$, where M is the representative matrix associated with the system, which depends on frequency ω and the Bloch vector K. Since the system of equations is homogeneous, a non-trivial solution can be obtained if the determinant of this matrix is zero. Thus it is possible to determine the band structure by finding the dispersion relation $\omega = \omega(K)$. For this, we define the determinant function:

$$D(K, \omega) = \ln(|\det(M(K, \omega))|). \quad (4)$$

Which presents local minimum points that allow to determine the eigenmodes of the system for a specific frequency.

3 Results

In numerical simulations it is common to introduce dimensionless values, so our results are expressed in terms of the a reduced Bloch vector given by $K_r = (P/2\pi)K$ and the reduced frequency $\omega_r = (P/2\pi)\,\omega$. Below we present some results of a PCW with an array of circular cylindrical inclusions involving a metamaterial under TM polarization (see Fig. 1). The following geometric values of the unit cell were taken into account: $b = 4\,\pi$, $P = 2\,\pi$ and $r = 0.1\,b$.

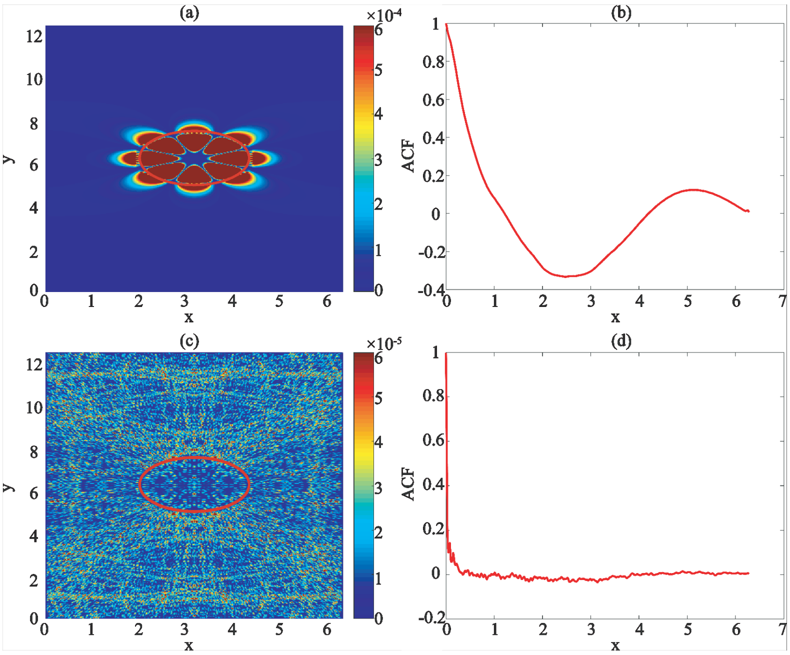

Fig. 2. TM-polarized electromagnetic intensity patterns in a unit cell of the PCW with LHM circular inclusions at the reduced frequencies (a) $\omega_r = 0.6488$ and (c) $\omega_r = 87.1359$. The solid circular curve represents the contour of the inclusion. The corresponding autocorrelation functions are shown in (b) and (d) with correlation lengths $\sigma = 0.2743$ and $\sigma = 0.0584$, respectively.

Figs. 2 (a) and (c), show the intensity of the magnetic field within the unit cell for the LHM inclusion for reduced frequencies $\omega_r = 0.6488$ and $\omega_r = 87.1359$, respectively. Each one with its respective natural frequency $\omega_0 = 0.6366$ and $\omega_0 = 63.6619$. For both cases, the corresponding autocorrelation functions (ACFs) were calculated (Figs. 2(b) and (d)) and their correlation lengths σ defined as the standard deviation of the autocorrelation function. For the lowest frequency, the correlation length obtained is $\sigma = 0.2743$ and for the highest frequency, a lower value $\sigma = 0.0584$.

We observe that the correlation length decreases as the field pattern is more disordered as the frequency increases. Furthermore, we believe this is a manifestation of electromagnetic wave chaos, since in this regime it led us to think that the intensity of the eigenmode is an uncorrelated random variable as a function of a point $(x,\ y)$ in the unit cell [3]. On the other hand, some classical systems with similar geometry, such the Sinai billiards [7] exhibit chaotic behavior.

4 Conclusions

An IEM was applied to study a PCW composed by two PEC flat surfaces and a periodic array of circular cylindrical inclusions of dispersive LHM. For certain conditions, disordered patterns of field intensities were obtained in our system.

In general, disordered patterns are associated with disordered systems, so this result, together with the fact that the corresponding classical model of our electromagnetic system presents a chaotic behavior, are our main arguments in terms of interpreting some of our results as manifestations of electromagnetic wave chaos in LHM media.

In addition, the signature of classical chaotic behavior can be seen in the spatial statistical properties of the system such as the correlation length. This assumption has been demonstrated in our results since the value of the correlation length is smaller when the frequency is high. These electromagnetic wave chaos properties in a PCW that include dispersive LHM media could have various applications in modern communication technology.

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